Numerical Analysis Saturation Profiles Compared with Analytical Solution of Buckley-Leverett Equation

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Abstract:

One of the simplest and most widely used methods of estimating the advance of a fluid displacement front in an immiscible displacement process is the Buckley-Leverett method. The Buckley-Leverett theory estimates the rate at which an injected water bank moves through a porous medium. The approach uses fractional flow theory and is based on the following assumptions and conditions: 1. D two-phase flow of incompressible fluids, e.g., Water displacing oil. 2. Oil and water are immiscible. 3. Homogeneous reservoir with constant properties. 4. Diffuse flow. 5. Gravity and capillary pressure effects are negligible. This method is well known; you almost always encounter this method when waterfloods are the topic of discussion. At many courses this method is taught as a general method for immiscible displacement, and then the interest normally stops. This study described a method for calculating saturation profiles when the effects of capillary pressure gradient and gravity are excluded. Based upon the solution of the basic partial differential equation, they found that, as time progresses, the saturation becomes a multiple-valued function of the distance coordinate X. Therefore, a numerical reservoir simulation model ECLIPSE® Simulation Software and analytical one has been developed for predicting the performance of two-phase fluid flow in a one-dimensional synthetic reservoir system. The validity of this synthetic reservoir model has been verified by comparing the solutions of numerical simulation with the analytical model from the Buckley-Leverett theory.

Keywords: Buckley–Leverett equation, Fractional flow, Immiscible displacement, Relative permeability, Reservoir simulation, Two-phase flow.

الملخص:

يُعد أسلوب باكلي–ليفرت أحد أبسط وأكثر الطرق استخدامًا في تقدير تقدّم جبهة الإزاحة في عمليات الإزاحة غير القابلة للامتزاج. حيث تقدّر نظرية باكلي–ليفرت المعدّل الذي تتحرك به جبهة الماء المحقون داخل الوسط المسامي، وتعتمد هذه الطريقة على نظرية التدفق الكسري وعلى مجموعة من الافتراضات والشروط، وهي: (1) تدفّق ثنائي الطور أحادي البُعد لسوائل غير قابلة للانضغاط، مثل إزاحة الماء للنفط، (2) عدم امتزاج النفط والماء، (3) خزان متجانس ذو خصائص ثابتة، (4) تدفق منتشر، (5) تأثيرات الجاذبية والضغط الشعري ضئيلة. هذه الطريقة معروفة جيدًا؛ وغالبًا ما تُصادفنا عند مناقشة تدفق المياه. تُدرّس هذه الطريقة كطريقة عامة للإزاحة غير القابلة

للامتزاج، دون اهتمام بها عادةً. وصــف هذا البحث طريقةً لحســاب أنماط التشـبع مع اسـتبعاد تأثيرات تدرج الضــغط الشعري والجاذبية. واستنادًا إلى حل المعادلة التفاضلية الجزئية الأساسية، وجد أنه مع مرور الوقت، يصبح التشبع دالة متعددة القيم لإحداثي المســافة X. لذلك، طُوّر نموذج محاكاة عددية للمكمن باســتخدام برنامج محاكاة ECLIPSE، وآخر تحليلي، للتنبؤ بأداء تدفق الســـوائل ثنائي الطور في نظام مكمن اصــــطناعي أحادي البعد. وقد تم التحقق من صحة هذا النموذج من خلال مقارنة حلول المحاكاة العددية بالنموذج التحليلي من نظرية باكلي–ليفرت.

الكلمات المفتاحية: التدفق ثنائي الطور، تدفق كســـري، الإزاحة اللاممتزجة، محاكاة المكامن، معادلة بوكلي–ليفيرت، النفاذية النسبية.

1. Introduction

In day-to-day business the reservoir simulator is normally used for even the simplest things. If you want to get a feeling of the performance of a waterflood, surfactant flood, or a steam flood almost all people immediately go to complicated models. However, the Buckley-Leverett method can in these situations also be used, it gives a good estimate of the best performance that will ever be observed. If you can't get it economical with these numbers, there is no need to try to do more complicated reservoir simulations since the performance will only get worse.

Most of the oil and gas recovered from reservoirs is displaced immiscibly by water and/or gas. The displacement could be in the form of solution gas drive, gas cap expansion, water influx from aquifers or injection of water and/or gas. Solution-gas drive, gas cap expansion, and water influx from aquifers are essentially natural processes that supply energy to the reservoir for hydrocarbon recovery. Gas and water injections are designed and installed to artificially supply energy to the reservoir and thereby improve hydrocarbon recovery.

It is important to understand the fundamental processes that occur when reservoir fluids are displaced immiscibly by gas or water. The displacement process is affected by the wettability of the rock, and the mobility ratio between the displaced and the displaced fluids. The total efficiency of the displacement process is measured in terms of the effectiveness of water or gas in displacing the reservoir fluids, and the proportion of the reservoir actually contacted by the displacing fluids.

In this chapter, basic concepts in immiscible fluid displacement are presented. These are then followed with the presentation of the fractional flow equation, the Buckley-Leverett[1] equation, and the Welge[12] method for estimating average water saturation in a water displacement process. These equations are presented to familiarize the engineer with some of the classical developments in the analysis of immiscible displacement processes before the advent and widespread application of reservoir simulation techniques. This approach is intended to enable the engineer to become conversant with some of the terms generally used in the industry to analyze and discuss the results from reservoir simulation when applied to immiscible displacement processes.

The objective of the research program reported herein is to develop and describe methods for reservoir simulation, including computer programs, to analyze two-phase fluid flow in a one – dimensional reservoir system. Moreover, several tests will be presented to verify the validity of the model.

The fundamental equations which are used to describe two-phase fluid flow in porous media include Darcy's Law for each phase. The special case of one – dimensional, incompressible, two-phase flow received much attention in the petroleum engineering literature in the early years. The basic about the displacement of oil by an injected fluid is that of Buckley and Leveret [1942][1]. This study described a method for calculating saturation profiles when the effects of capillary pressure gradient and gravity are excluded.

Based upon the solution of the basic partial differential equation, they found that, as time progresses, the saturation becomes a multiple-valued function of the distance coordinate X. Later, the one-dimensional displacement equation for a homogeneous permeable porous medium, including the effects of capillary pressure and gravity forces, were solved[2].

Most of the mathematical derivations of equations used in the numerical simulation herein are adaptations and extensions of previous work on one-dimensional and two-phase flow. This method excludes the effects of capillary pressure and gravity forces. However, the compressibility of both oil and water is considered. In addition, pressure profiles are calculated implicitly, and saturation profiles are calculated explicitly. Relative permeabilities required are functions of saturation only.

2. Literature Review

Considerable progress has been made in the past few years for obtaining numerical solutions of equations concerning two-phase flow and multiphase flow in porous media. Several papers appeared in the literature describing the quantitative treatment of waterflood recovery problems. A useful purpose may be served herein by outlining some of these significant papers.

Buckley and Leverett [1942][1] established a theory of oil displacement based on the relative permeability concept. They described the mechanism by which the displacement occurred. Moreover, a method for calculating saturation profiles was developed when the effects of capillary pressure gradient were ignored. In their original solution of the two-phase flow problem, Buckley and Leverett observed that the solution of the two-phase flow equations became multiple-valued in saturation, even though it is physically unrealistic for saturation to have more than one value at a given position.

The Buckley and Leverett [1942][1] analysis is the first pioneer work in the study of linear displacement of a fluid by another fluid. The solution of their displacement study on two-phase fluid excluded the effect of capillary and gave multiple results for saturation at a given position. Holmgren and Morse [1951][3] utilized the Buckley-Leverett theory to calculate the average water saturation at breakthrough and explained dispersion because of capillary effects.

West, Garvin, and Sheldon [1954][4] presented a general discussion of the two-phase flow problem and treated linear and radial systems with both capillary pressure and gravity with consideration of the effects of compressibility.

Douglas, Blair, and Wagner [1958][5] developed different methods for solving the one-dimensional case for incompressible fluids with capillarity using finite difference methods.

To solve the displacement equation including capillary as well as gravity, Fayers and Sheldon [1959][6] failed to determine the time required to obtain a particular saturation, which later was explained by Bentsen [1978] revealing the fact that the distance traveled by zero saturation is governed by a separate equation.

Bentsen also noted that at slower injection rates, the input boundary condition of constant normalized saturation that Fayers and Sheldon [1959][6] used was incorrect in formulation. Also, there have been numerical investigations in the past to solve the displacement equation.

3. Study Objective

This study is directed towards to perform a comparison between the saturation profiles from the numerical solution by using **ECLIPSE® Simulation Software**[7] and the analytical Solution of Buckley Leverett Equation, excluding the effect of capillary forces in both imbibition and drainage process.

In addition, understanding the multiphase flow behavior in waterflood mechanisms and investigate the factors that control the front displacement.

4. Methodology

There are two techniques for solving mathematical reservoir models: analytical and numerical. Each of these has certain strengths and limitations.

4.1. Analytical Techniques

Analytical or closed-form techniques offer the advantage of providing exact solutions (when they can be found); furthermore, those solutions are continuous throughout the system. The types of problems that are amenable to analytical solution, however, are very limited. Analytical methods fall short when we start dealing with varying formation thickness, non-uniform porosity and permeability, and changing fluid properties, and other such conditions that describe most real reservoirs. To find analytical solutions for the type of system that "Mother Nature" generally provides, we have to modify the problem – sometimes quite drastically – to make it plausible for handling analytically. What we end up doing is providing an exact solution to an approximate problem (e.g., a classical well test analysis model).

4.2. Numerical Solution

A numerical solution involves discretizing, or approximating the mathematical model - that is, using a numerical tool such that continuous forms of the partial differential equations are written in a discrete form. We perform this discretization process not only on the partial differential equation, but also on the physical systems. This means that we divide the physical system into a number of sub-domains that are coupled to one another.

The clear advantage of the numerical approach is that it allows us to assign representative properties to as many parts of a system as we have information for. However, we must not forget that we inevitably lose some measure of accuracy in discretizing the partial differential equations. The net result is that in using a numerical approach, we are providing an approximate solution to an exact problem.

5. Analytical Techniques

Most of the analytical methods estimate volume of cumulative oil recovery as a function of cumulative water injection. They do so by:

1. Dividing the total reservoir thickness into the desired number of distinct permeability layers (by various approaches) or flow zones (current approach), making sure that the vertical permeability distribution is correctly mimicked.
2. Allocating the injected water rate at any time among the flow zones.
3. Calculating cumulative oil recovery as a function of cumulative water injection into each flow zone.
4. At various times during the flood life, combining the zonal oil recovery and water injection to obtain performance of the total reservoir.
5. Relating oil recovery to time by using the average injection rate estimated during that time step.

Craig [1971][9] has described all the methods and their strengths / shortcomings in his SPE monograph. The biggest advantage of these analytical methods is that one can predict the composite (gross) behavior - oil recovery, water requirements, water-cut - of the reservoir. The biggest shortcoming is that they cannot predict the details on sector (areal or vertical) or individual well basis.

These methods include:

1. 1-D Models for estimating Displacement Efficiency, E_D
    - Buckley & Leverett method for Dispersed Flow.
    - Dietz method for Segregated Flow.
2. 2-D Areal Models for estimating Areal Sweep Efficiency, E_A
3. Layered Models for estimating Vertical Sweep Efficiency, E_V

- Stile's method.
- Dykstra and Parson's method.

4. Many methods incorporating all the above.

These methods are dated and are not often used these days for detailed analysis. However, they were used extensively prior to the availability of reservoir simulation models.

The use of these methods is still recommended, as they provide:

- Insights into the mechanisms (physics) of the WF process.
- Approximate recovery estimates which could serve to judge the credibility of the results of a simulation study.

5.1. BUCKLEY LEVERET (Basic Fractional Flow Procedure)

The Buckley-Leverett [1958][10] equation is based on the principle of conservation of mass for linear flow of a fluid (water or gas) through a reservoir at constant total flow rate. It is also the most common analytical procedures are tied to the Fractional Flow relationships.

To illustrate the derivation of the Buckley-Leverett equation, the case of water displacing oil is used. Note that the same equation can be developed representing the case for gas displacing oil.

The most common basis for these procedures was developed by Buckley & Leverett as follows:

- How much oil and water are flowing at any point of interest in the reservoir
- How long does it take for the injected water to reach a point of interest (Producer)
- How does displacement efficiency relate to injection volume?

Consider a volume element of a linear reservoir model shown in Figure [3.1]. Let the thickness of the element be represented as □x and located at a distance, x, from the inlet face of the linear model.

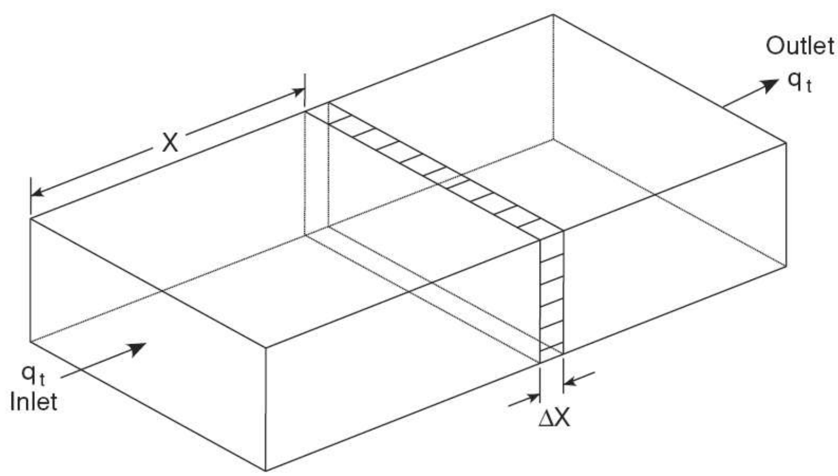


Figure (1): Linear reservoir model.[11]

A volumetric balance in terms of the water phase (assuming density of water is constant) for the element of the reservoir model can be written as:

$$\begin{bmatrix} Vol. of\ water\ flowing \\ into\ element\ in\ time, \Delta t \end{bmatrix} - \begin{bmatrix} Vol. of\ water\ flowing \\ out\ of element in\ time, \Delta t \end{bmatrix} = \begin{bmatrix} Accumulation\ of\ water \\ in\ element\ time, \Delta t \end{bmatrix}$$

The previous expression can be expressed algebraically as:

$$[f_w\, q_t\, \Delta t]_x - [f_w\, q_t\, \Delta t]_{x+\Delta x} = \left[\frac{\varphi\, A\, \Delta x}{5.615} \cdot \Delta S_w\right] \quad \text{.......................................} \quad (1)$$

Re-arranging Eq. [1] gives:

$$\left[\frac{\varphi\, A\, \Delta x}{5.615\, q_t} \cdot \frac{\Delta S_w}{\Delta t}\right] = -\frac{[f_w]_{x+\Delta x} - [f_w]_x}{\Delta x} \quad \text{..} \quad (2)$$

Taking limits as $\Delta t \rightarrow 0$ and $\Delta x \rightarrow 0$ yields the continuity equation:

$$\left(\frac{\varphi\, A}{5.615\, q_t}\right) \cdot \left(\frac{\partial S_w}{\partial t}\right)_x = -\left(\frac{\partial f_w}{\partial x}\right)_t \quad \text{..} \quad (2)$$

As stated, the fractional flow of water is a function of water saturation only if fluid properties and total flow rate are constant. By application of chain rule $f_w = f_w(S_w)$ can be expressed as:

$$\left(\frac{\partial f_w}{\partial x}\right)_t = \left(\frac{\partial f_w}{\partial S_w}\right)_t \cdot \left(\frac{\partial S_w}{\partial x}\right)_t \quad \text{..} \quad (3)$$

Substituting Eq. [4] into Eq. [3] and re-arranging gives:

$$\left[\frac{\partial S_w}{\partial t}\right]_x = -\frac{5.615\, q_t}{\varphi\, A}\left[\left(\frac{\partial f_w}{\partial S_w}\right)_t \left(\frac{\partial S_w}{\partial x}\right)_t\right] \quad \text{..} \quad (4)$$

Equation [5] gives water saturation as a function of time at a given location. A more useful equation expressing water saturation as a function of location at a given time can be developed from Eq.[5]. For any displacement, the distribution of water saturation is a function of both location and time. This is represented as:

$$S_w = S_w(x,t) \quad \text{...} \quad (5)$$

The total derivative of is then:

$$dS_w = \left(\frac{\partial S_w}{\partial x}\right)_t dx + \left(\frac{\partial S_w}{\partial t}\right)_x dt \quad \text{...} \quad (6)$$

Since the focus is on a fixed water saturation, then dS_w=0 . And Eq. [7] becomes:

$$0 = \left(\frac{\partial S_w}{\partial x}\right)_t dx + \left(\frac{\partial S_w}{\partial t}\right)_x dt \quad \text{.......} \quad (7)$$

By re-arrangement, Eq. [8] becomes:

$$\left(\frac{dx}{dt}\right)_{S_w} = -\frac{\left(\frac{\partial S_w}{\partial t}\right)_x}{\left(\frac{\partial S_w}{\partial x}\right)_t} \quad \text{.......} \quad (8)$$

$$\left(\frac{dx}{dt}\right)_{S_w} = -\frac{5.615\, q_t}{\varphi A}\left(\frac{\partial f_w}{\partial S_w}\right)_t \quad \text{.......} \quad (9)$$

Since the total flow rate is assumed to be constant, then fractional flow of water is independent of time. Hence,

$$\left(\frac{\partial f_w}{\partial S_w}\right)_t = \frac{df_w}{dS_w} \quad \text{.......} \quad (10)$$

Equation [11] then becomes:

$$\left(\frac{dx}{dt}\right)_{S_w} = \frac{5.615\, q_t}{\varphi A}\frac{df_w}{dS_w} \quad \text{.......} \quad (11)$$

Equation [12] is the Buckley-Leverett equation. It is also called the frontal advance equation. Integration of Eq. [12] yields a useful form of the Buckley-Leverett equation:

$$x = \frac{5.615\, q_t\, t}{\varphi A}\left(\frac{df_w}{dS_w}\right)_{S_w} \quad \text{.......} \quad (12)$$

Equation [13] can be used to calculate the distribution of water saturation as a function of time in a linear reservoir under water injection or aquifer influx. The distance travelled by a given saturation in a specified time interval is proportional to the slope of the fractional flow curve at that saturation assuming the total flow rate and reservoir properties are constant. Using this approach, the distribution of water saturation in the reservoir as a function of time can be calculated by determining the slope of the fractional flow curve at that saturation. However, because of the shape of the fractional flow curve, it is possible that two slopes of equal value can exist for two different water saturations. Appling Eq. [13], this is interpreted to indicate that two different water saturations can exist at the same location in the reservoir at the same time. The appearance of this contradiction in the application of the frontal advance equation is illustrated in Figure [2].

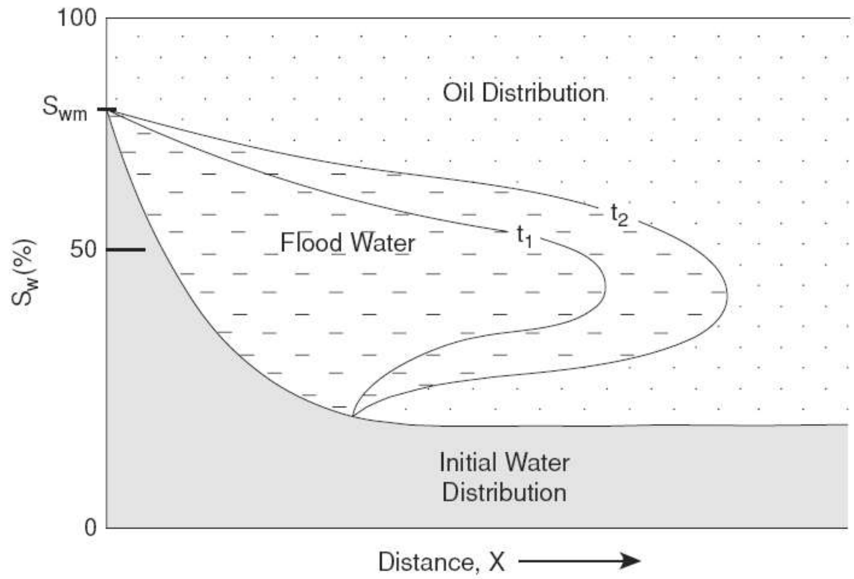


Figure (2): Saturation distribution based on frontal advance equation.[11]

Buckley and Leverett [References] recognized that a portion of the saturation distribution curve is imaginary and that the real curve is discontinuous at the flood front. The location of the flood front as determined by material balance is represented in Figure [3] by a solid line such that areas A and B are equal. Note the sharp discontinuity of the saturation curve at the flood front as represented in Figure [3]. This is because capillary and gravity effects were assumed to be negligible. If capillary and gravity effects are considered, the distribution of water saturation at the flood front is more gradual as represented in Figure [4].

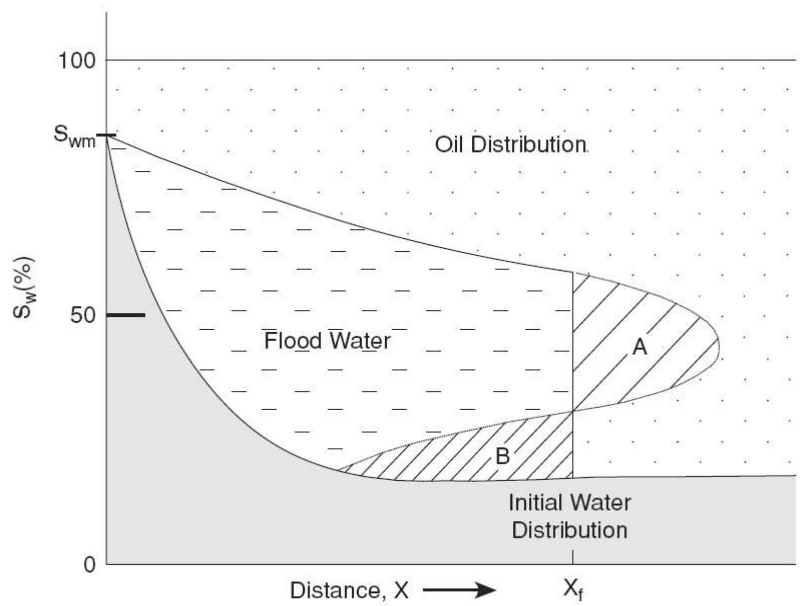


Figure 0): Location of the flood front as determined by material balance.[11]

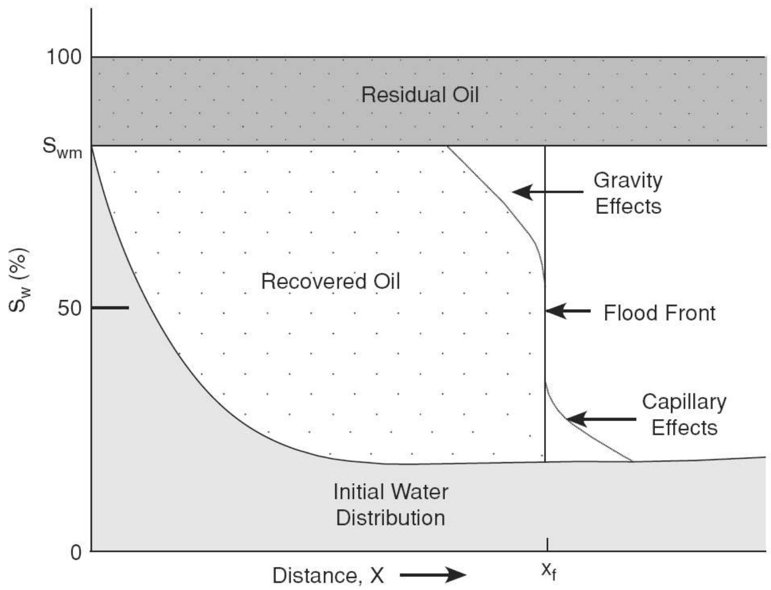


Figure (4): Location of the flood front with capillary and gravity effects. [11]

5.2. The Welge Method[12]

Welge [1952][12] proposed a method for computing oil recovery from gas or water drive that simplified the application of the Buckley-Leverett method. The Welge [1952][12] method is presented with graphical illustrations for the case of water drive in a linear reservoir. The graphical illustrations can be replicated for gas drive by simply replacing with respectively.

5.3. Water Saturation at the Flood Front

Water saturation at the flood front, can be determined graphically using the Welge method by drawing a straight line from initial water saturation tangent to the fractional flow curve as shown in Figure [5].

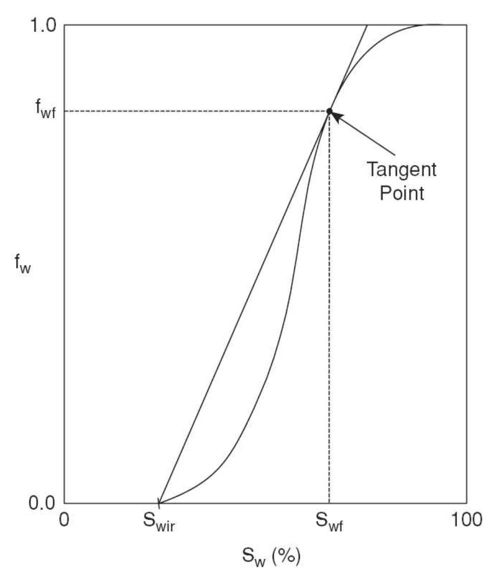


Figure (5): Fractional flow curve with application of the Welge method.

If the initial water saturation is greater than the irreducible water saturation, the tangent line is drawn from the initial water saturation as shown in Figure [6].

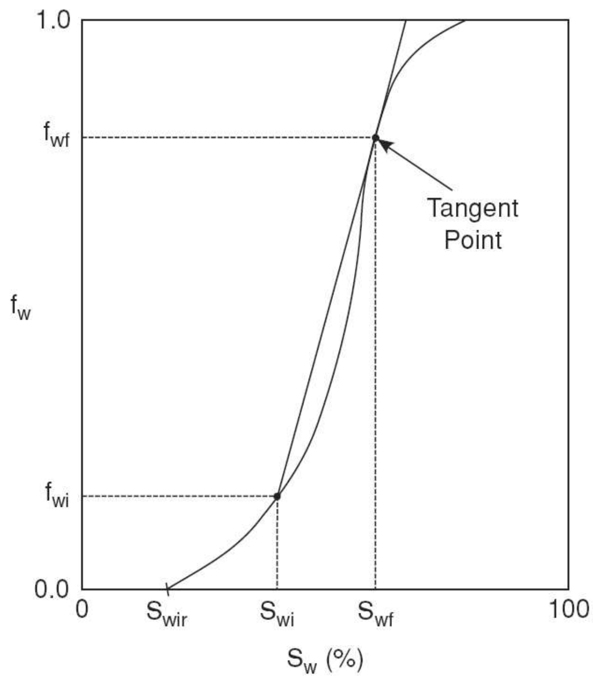


Figure (6): Fractional flow curve with application of Welge method for Swi > Swir.

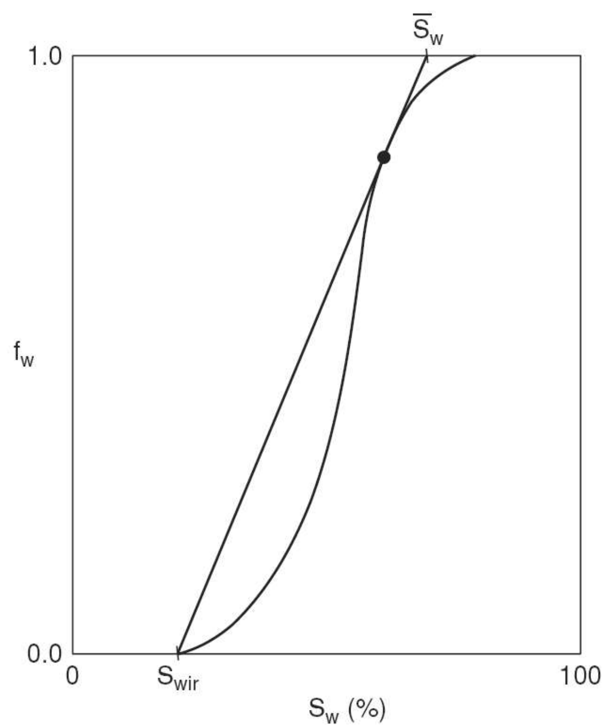


Figure (7): Fractional flow curve with application of Welge method for average water saturation.

5.4. Average Water Saturation behind the Flood Front

By extending the tangent line drawn to the fractional flow curve as shown in Figures [5] or [6] to the point where f_w=1.0, the average water saturation $\overline{S}_w$, behind the flood front can be determined as shown in Figure [7]. At water breakthrough, $\overline{S}_w = \overline{S}_{wbt}$, where $\overline{S}_{wbt}$ is the average water saturation in the reservoir at water breakthrough.

5.5. Average Water Saturation after Water Breakthrough

The average water saturation after water breakthrough is determined as shown in Figure [8] by drawing a tangent line to the fractional flow curve at water saturation, S_{w2}, greater than but less than the maximum water saturation, . The water saturation, is the saturation at the outlet end of the linear system after water breakthrough with the corresponding fractional flow of water denoted as. By extending the tangent line to the point where, the average water saturation, in the system after water breakthrough is determined.

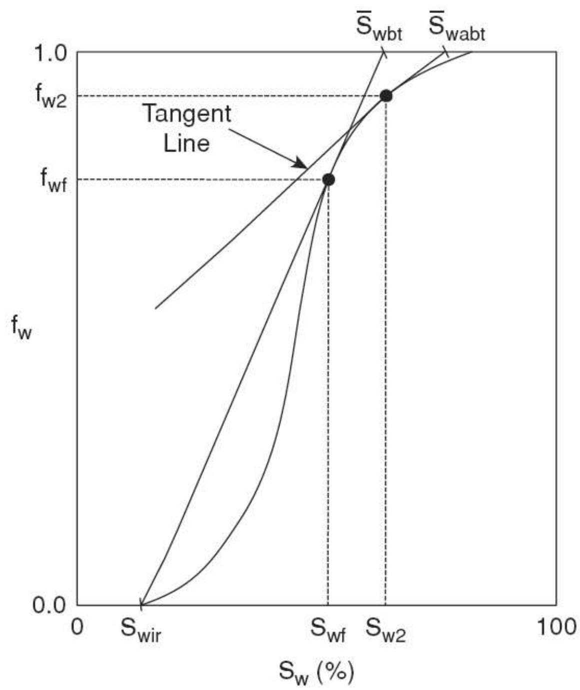


Figure (8): Application of Welge method for average water saturation after water breakthrough.

5.6. Fractional Flow Equation

The fractional flow equation is used to calculate the flow rate of a fluid as a fraction of the total fluid flow rate when only two fluids are flowing in the reservoir. The flow rate of the fluid at any point in the reservoir depends on its saturation at that point. Since relative permeability of the fluid is dependent on saturation, it follows then that the flow rate of the fluid is dependent on its relative permeability at that point in the reservoir. Fractional flow of a fluid in a reservoir is primarily dependent on its relative permeability but can be affected by capillary and gravity forces. The fractional flow equations developed here assume of linear flow.

The fractional flow equation developed in this section is for water displacing oil in an oil-water reservoir.

5.7. Assumptions

The following assumptions are made:

1. The system is linear, horizontal and of constant thickness.
2. The flow is isothermal, incompressible, linear and obeys Darcy's law.
3. Capillary and gravity forces are negligible.
4. The system is only one homogeneous layer with uniform thickness and constant permeability.
5. The relative permeability characteristics are the same for all layers.
6. The initial fluid saturation is uniform at the irreducible water saturation.
7. The porosity is assumed constant.

A combination of the above is utilized for estimating the above-mentioned information. Welge's simplified, graphical solution will be presented here.

5.8. Fractional Flow of Water Displacing Oil

$$f_w = \frac{1 + 0.001127\,\frac{k\,k_{ro}}{\mu_o}\,\frac{A}{q_t}\left[\frac{\partial P_c}{\partial L} - 0.433\,\Delta\rho\,\sin\alpha_d\right]}{1 + \frac{\mu_w}{\mu_o}\,\frac{k_{ro}}{k_{rw}}} \qquad (13)$$

$f_w =$

$$\frac{1}{1 + \frac{\mu_w}{\mu_o}\,\frac{k_{ro}}{k_{rw}}}$$

Viscous Force

$$\frac{0.001127\,\frac{k\,k_{ro}}{\mu_o}\,\frac{A}{q_t}\left[\frac{\partial P_c}{\partial L}\right]}{1 + \frac{\mu_w}{\mu_o}\,\frac{k_{ro}}{k_{rw}}}$$

Capillarity Force

$$\frac{0.001127\,\frac{k\,k_{ro}}{\mu_o}\,\frac{A}{q_t}\left[0.433\,\Delta\rho\,\sin\alpha_d\right]}{1 + \frac{\mu_w}{\mu_o}\,\frac{k_{ro}}{k_{rw}}}$$

Gravitational Force

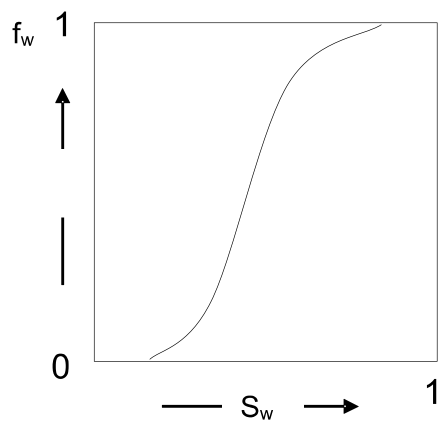
4.9. Frontal Advance Equation

This equation relates the rate of advance of a known saturation to the total fluid velocity and to the change of fractional flow caused by a small change in the saturation of water.

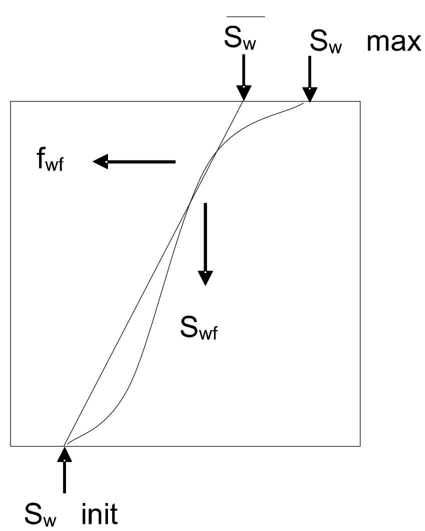
$$x\Big|_{Sw} = \frac{5.615 \cdot Q_i \cdot \Delta t}{\varphi A}\left[\frac{df_w}{dS_w}\right]_{Sw} \quad \text{...} \quad (14)$$

5.10. Procedure for Buckley-Leverett Method for Waterflood Prediction

- **<u>Step No. 1:</u>** Calculate f_w as a function of S_w using the equation appropriate for the situation and plot it on Cartesian paper.



- **<u>Step No. 2:</u>** Draw tangent to $f_w - S_w$ curve from $S_{w\text{-}init}$ value of S_w at initiation of waterflood. Look up values of S_{wf}, f_{wf} and S_w.

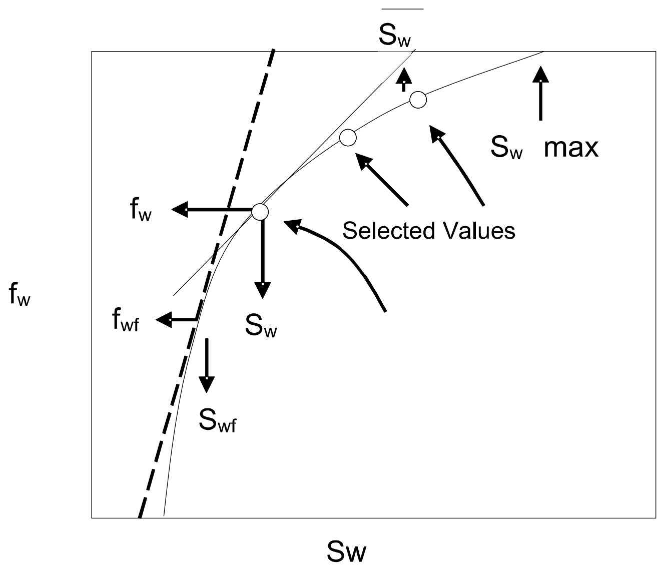


Calculate the slope of the tangent drawn from $S_{w\text{-}init}$.

$$f_w' = \left[\frac{df_w}{dS_w}\right] = \frac{\Delta f_w}{\Delta S_w} = \frac{1}{\left(Sw - Sw_{init}\right)}$$

- **Step No. 3:** Select 6 or 7 values of S_w in between S_{wf} and $S_{w\text{-}max}$. Draw tangents to $f_w - S_w$ plot from each of the selected S_w value. Look up corresponding values of f'_w and S_w at each point. Calculate slope of the tangents at each of the selected S_w value.

$$f_w' = \frac{\left(1 - f_w\right)}{\left(Sw - \overline{Sw}\right)}$$



6. Numerical Solution

6.1. General Structure of Reservoir Flow Models

The structures of input data in most reservoir flow simulators are remarkably similar. The general structure of reservoir simulation models is described here to acquaint the reader with sequence of data input in most simulators. Obviously, the structure of input data will vary slightly between different simulators. But in many cases, the differences are minor and can be quickly reconciled between simulators from different sources. The purpose of this section is to familiarize the reader with a readily available data structure that can be used to transfer data from one simulator to the other.

6.2. Definition of Model and Simulator

The entry of data into every simulator begins with definition of the size of the reservoir model and the type of simulator to be used for modeling the reservoir. The key entry that defines the size of the reservoir model is the number of grid blocks in the model. For instance, in the Cartesian system, the number of grid blocks in the directions for a 3D model is specified. Also defined in this data entry section is the number of wells in the model, the number of tabular data (PVT, relative permeability, equilibration regions, etc.), and the number of initialization regions, etc. The type of simulator to be used including the formulation (solution) type is specified in this section.

For instance, a black oil model based on the Implicit formulation may be selected. Most important, the date for the start of simulation is specified. This section may be considered as the section in which the scope of the simulation problem is specified for the simulator. In many ways, this section defines the amount of computer memory that will be required to run the reservoir model.

Our main concern (about the reservoir fluid model) was to select a simulator that best represents the diffusion phenomena. In order to accomplish the objectives of this thesis; ECLIPSE 100 (**Finite Difference Numerical Simulator**) was used.

6.3. Model Description

A synthetic linear reservoir model with a single injector and producer wells, the injector is located at the first while the producer is located at the last cell in the X-direction. The wells were assumed to be perforated across the height of the reservoir, Figure [9]. The bottom-hole flowing pressure and production rates are specified.

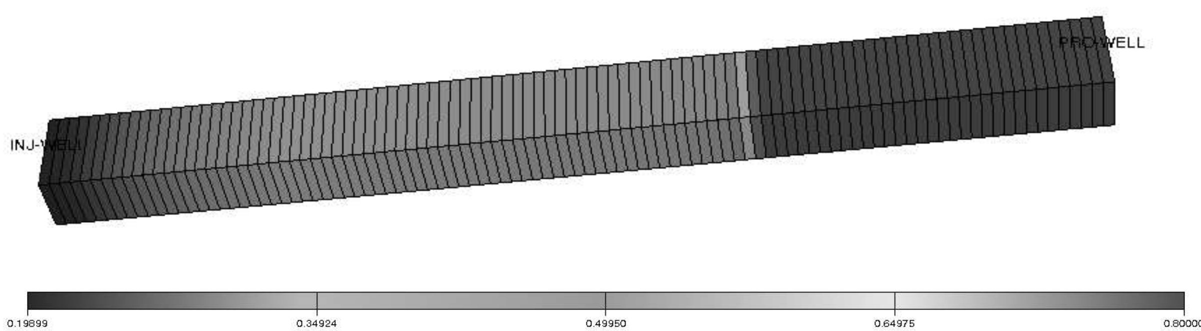


Figure (9): Schematic of the linear homogenous grid model.

6.4. Geologic Model Data

All the structural and petrophysical data in the geological model are typically assembled as data input for the gridblocks in a section of the simulator. The structure of the geologic model is represented by geometrical data on the gridblocks in terms of location and dimensions. This is usually accompanied with separate specifications of the petrophysical data for each gridblock. The petrophysical data usually specified for each gridblock include porosity, permeability, and net sand thickness or net-to-gross ratio data. Initial fluid saturations for each gridblock may also be specified in some models. These data are then followed with modifications to the grid system (such as local grid refinement), and modifications to the petrophysical data specified for the gridblocks.

All cells have a uniform thickness of 1m. The assigned absolute permeability was 1 Darcy and the porosity of 0.20 (fraction). The simulation case was generated from an initial reservoir pressure above the bubble-point pressure of the selected fluid, which means that, initially, the only fluid in the reservoir was oil. Table [1] presents the reservoir properties.

Table (1): Basic reservoir parameters used in the compositional simulation model.

Water Compressibility, c_w, psi^{-1}	**3.0 x 10^{-6}**
Oil Compressibility, c_o, psi^{-1}	3.0 x 10^{-6}
Rock Compressibility, c_r, psi^{-1}	6.0 x 10^{-6}
Core height, h, meter	1
Porosity, ∅, %	20
Absolute Permeability, k, md	1000
Core width, W, meter	1
Irreducible Water Saturation, S_{wi}, %	20
Reference Pressure, bar	1
Residual Oil Saturation, S_{or}, %	15

6.5. Fluid Properties Data

This section of the model data set contains data that represents the PVT properties of the fluids present in the reservoir. The PVT data are usually presented in a tabular form for black oil models. For compositional simulators, the PVT data are represented in a compatible form as output generated with an equation of state.

Table (2): Basic PVT parameters used in the compositional simulation model.

Water Formation Volume Factor, B_w, m^3/Sm^3	**1**
Oil Formation Volume Factor, B_o, m^3/Sm^3	1.1
Water Viscosity, $\square_w$, Pa s	0.001
Oil Viscosity, $\square_o$, Pa s	0.004
Water Density, $\square_w$, kg/m^3	1000
Oil Density, $\square_o$, kg/m^3	400

6.6. Rock/Fluid Properties Data

Rock/fluid properties data in the form of relative permeability data and capillary pressure data are represented in the model as functions of fluid saturations. These data are usually presented in the simulator in a tabular form. Note that the data in these tables are sometimes used by the simulator to establish initial conditions in the reservoir model, if the option for simulator generated initial conditions is selected.

6.7. Relative Permeability Model

The relative permeability is an important input parameter for reservoir simulation studies and provides a basic description of the movement of phases in the reservoir. It is also used to describe multiphase flow in a porous media. A change in relative permeability has a significant effect on the predicted hydrocarbon production rates and the overall recovery factor.

Typical curves suitable for an oil-water system with water displacing oil are presented in Figure [10]. The value of S_w at which water starts to flow is termed the critical saturation, S_{wc}, and the value at which oil ceases to flow, S_{nc}, is called the residual saturation. Analogously, during a drainage cycle S_{nc} and S_{wc} are referred to as the critical and residual saturations, respectively.

Two phase relative permeability is often represented with simple models such as the Brooks-Corey model[13] and with the parameters end-point relative permeability and residual fluid saturations. The residual oil saturation is clearly by far the most important of these parameters and deserves special attention.

The relative permeabilities were estimated for a specific initial water saturation using the standard Corey expressions following Liu et al. [2001][13] the resulting relative permeability curves and the Corey function parameters are shown in Figure [10].

$$k_{ro} = k_{ro}^{0}\left(\frac{1-S_w-S_{or}}{1-S_{wc}-S_{or}}\right)^{n_o} \quad \text{...} \ (15)$$

$$k_{rw} = k_{rw}^{0}\left(\frac{S_w-S_{wc}}{1-S_{wc}-S_{or}}\right)^{n_g} \quad \text{..} \ (16)$$

Where n_o and n_w are exponents of oil and water respectively.

Table [3], lists the relative permeability end-point values used in the Corey's functions of the base case.

Table (3): Relative permeability end-point parameters.

S_{wi}	S_{or}	k_{romax}	k_{rwmax}	n_o	n_w
0.20	**0.15**	**1.0**	**1.0**	**2.5**	**4**

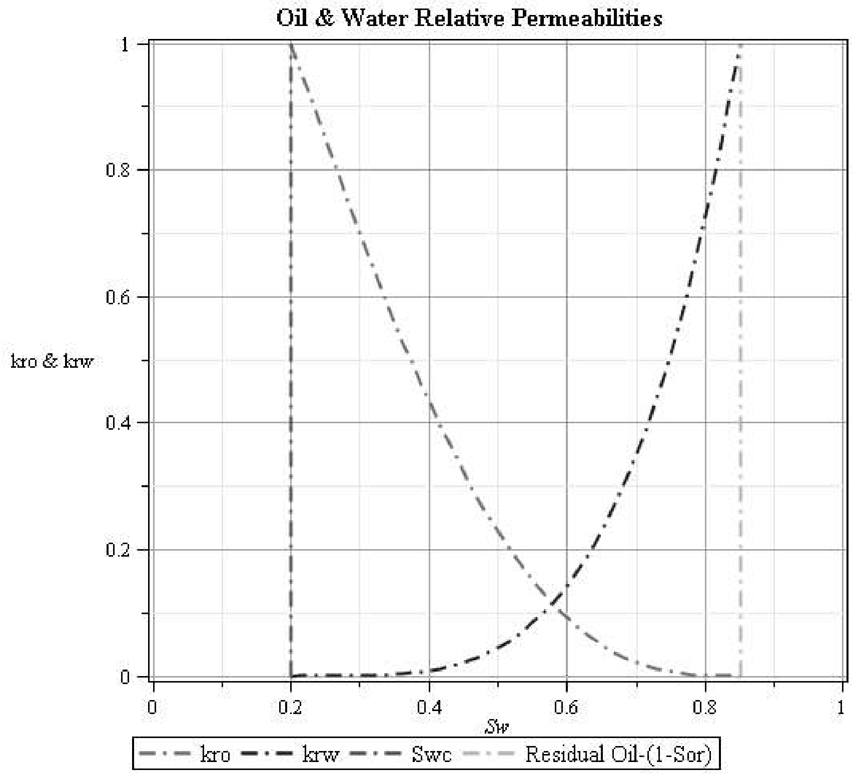


Figure (10): Brooks-Corey relative permeabilities with different critical condensate saturations.

6.8. Model Equilibration Data

Model equilibration data include fluid contact depths (oil-water contact, gas-oil contact, or gas-water contact), capillary pressures at the fluid contacts, and reservoir pressure at a selected datum depth. The model equilibration data are in some cases used by the simulator to establish initial reservoir conditions.

6.9. Well Data

In the section of the model data for wells, the locations of the wells in the grid system are specified. Also, the grid blocks in which the wells are completed are specified. The production or injection rates of the wells including the type of fluid produced or injected are specified. The progression of the simulation in terms of time is defined in this section in the form of time steps, cumulative time, or dates. These time-based data are very important because the speed and duration of the simulation are controlled by these data. For reservoirs with production history, the production data are provided at specific time intervals which could be daily, monthly, quarterly, semi-annually, or annually. The frequency of production data entry is totally at the discretion of the user. However, note that higher frequency of production data specifications reduces the speed of the simulator during history match. Additional data that may be specified at progressive time periods include introduction of new wells, recompletion of existing wells, and changes to well fluid production or injection rates.

6.10. Initialization

The reference pressure is 1 bar. The injection well considered in this study is operating with a constraint of injection rate of 0.2 m^3/day. The producer in this simulation was controlled by an oil rate of 0.2 m^3/day. The well was initially produced at the designated oil rate and stooped to control when the oil rate dropped below the oil rate minimum limit.

6.11. Assumptions in the model:

A number of assumptions and simplifications have been made in order to make the attempt to study the problem as following:

- Gravitational segregation of the condensate is not considered.
- No compositional gradient is considered.
- No irreducible water saturation.

7. Results and Discussion

To test the validity of the numerical solutions for the simulation model, it is necessary to use data taken under the same conditions into computations using other methods.

In general, the one-dimensional, two-phase characteristics of the reservoir flow are considered. Saturation is a function of reservoir pressure and flow distance. Relative permeability is a function of saturation only.

7.1. Presentation of Analytical Solution

According to the Buckley-Leverett theory, water saturation is a function of both time and position X. We have found that the water saturation at water (front) breakthrough and also the average water saturation in the reservoir after water breakthrough.

A typical plot of variation of relative permeability to water, k_{rw}, relative permeability to oil, k_{ro}, fractional flow curve, f_w and its derivative, df_w/dS_w are shown in Figure [11] to [13]. The capillary pressure data are neglected and assumed to be zero.

We need to construct a straight line from the (initial) connate water saturation (f_w=0) that is tangent to the fractional flow curve.

The point where this straight line and the fractional flow curve meet gives the water saturation at waterfront breakthrough. Extrapolation of the line till f_w = 1 gives the average water saturation behind the waterfront.

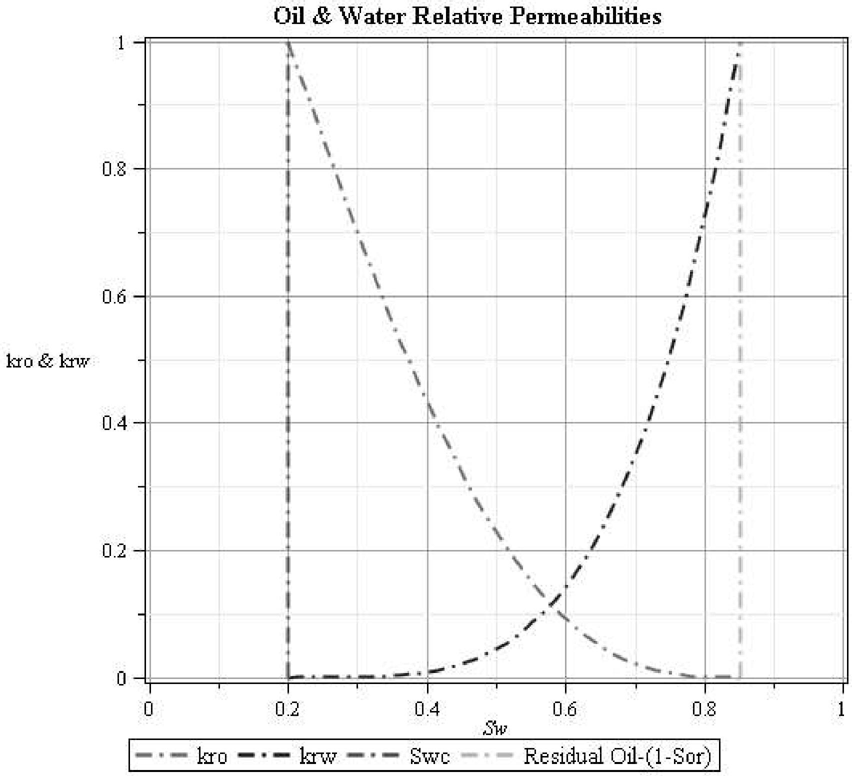


Figure (13): Relative permeabilities curves versus water saturation.

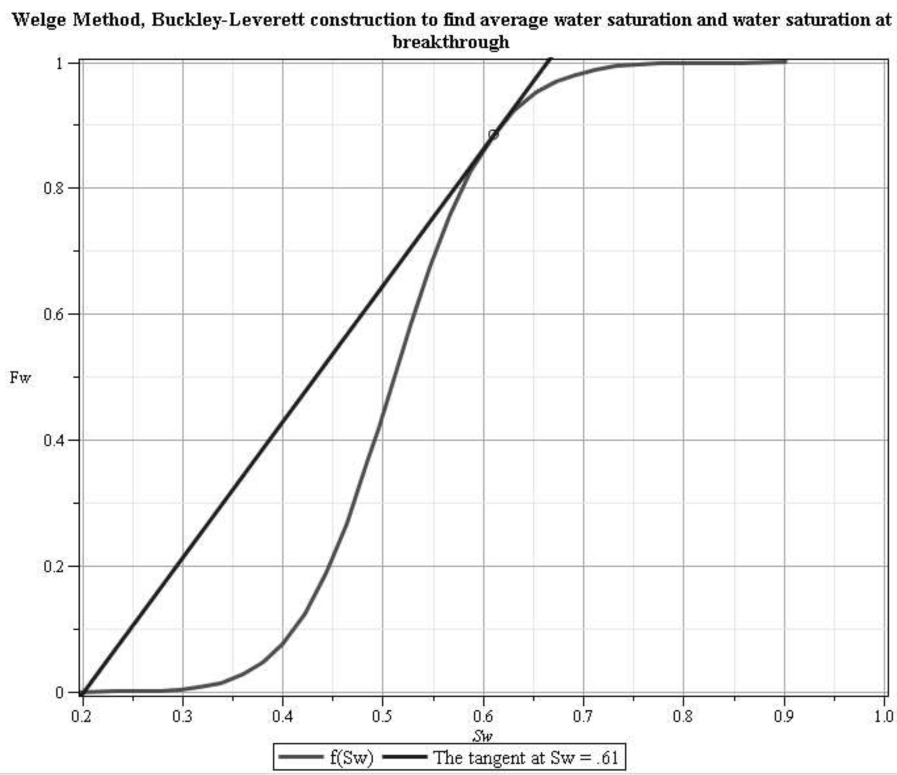


Figure (12): Buckley-Leverett construction to find average water saturation and water saturation at breakthrough.

Graphically this is very simple to do, but in a spreadsheet the slope of the straight line and the fractional flow curve have to be calculated and compared. The slope of the fractional flow curve is calculated with:

$$\frac{df_w}{dS_w} = \frac{\frac{1}{\mu_w}\frac{dk_{rw}(S_w)}{dS_w}(u_w+u_o) - u_w\left(\frac{1}{\mu_w}\frac{dk_{rw}(S_w)}{dS_w} + \frac{1}{\mu_o}\frac{dk_{ro}(S_w)}{dS_w}\right)}{(u_w+u_o)^2} \quad \text{.................} \quad (18)$$

With,

$$\frac{dk_{rw}(S_w)}{dS_w} = \frac{1}{1-S_{wc}-S_{or}}\left(\frac{S_w-S_{wc}}{1-S_{wc}-S_{or}}\right)^{n_w-1} n_w\, k_{rw}(S_{or}) \text{........................} \quad (19)$$

$$\frac{dk_{ro}(S_w)}{dS_w} = \frac{1}{1-S_{wc}-S_{or}}\left(\frac{1-S_w-S_{or}}{1-S_{wc}-S_{or}}\right)^{n_o-1} n_o\, k_{ro}(S_{wc}) \text{....................................} \quad (20)$$

The slope of the straight line can be easily calculated. When both derivatives are plotted in one single graph the tangent point is found.

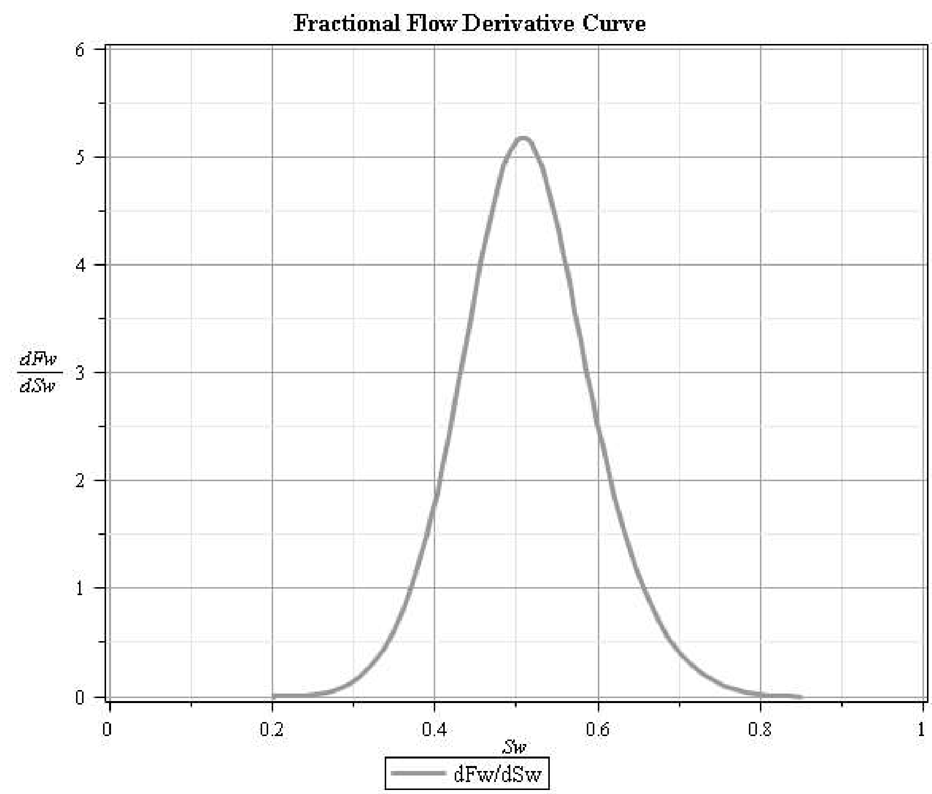


Figure (13): Derivatives of fractional flow curve and tangent line.

The distribution of the water saturation along the reservoir is given in Figure [14]. The results of the water saturation without the effect of the capillary pressure are depicted in the same Figure.

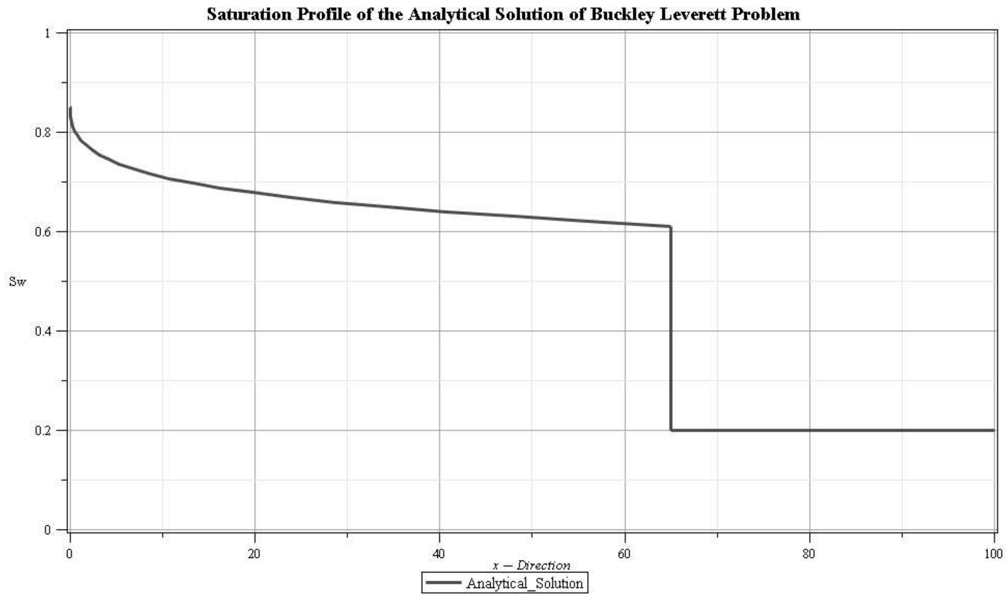


Figure (14): Saturation profile calculated from Buckley-Leverett analysis

7.2. Presentation of Numerical Calculation Results

To compare numerical solutions with the Buckley-Leverett results, a numerical simulation model involving water displacing oil from a water-wet porous medium is presented with the same considerations as in the Buckley-Leverett displacement theories. Moreover, compressibilities of oil and water in the reservoir are also included in this simulation model. The initial conditions include uniform saturation and pressure distributions. The grid spacing (Ax) for the numerical simulation is constant. It was noted that the saturation profiles computed under these assumptions show water accumulating at the downstream end of the system, while the fluid front proper is still traveling toward the outlet. The results based on the simulation computation model are shown in the forms of Saturation profiles within the hypothetical reservoir versus distance (along the X-direction).

The behavior of the linear, homogeneous reservoir is determined using the reservoir simulator. To maintain the desired pressure in a reservoir, for any volume of water injected into the upstream end of the system, an equivalent volume of oil is produced at the other end. The fluid properties are functions of pressure for the model in this report. Viscosity is the dominant factor in the system. The effects of capillary and gravity forces are negligible and are, therefore, ignored. The initial conditions of the reservoir were taken to be 80 percent oil saturation and 20 percent water saturation.

Assumptions for the numerical model described in Chapter 3 are consistent with those for the Buckley-Leverett analysis. The results calculated from the Buckley-Leverett technique are presented in the Chapter. Consequently, the saturation profiles and all the results calculated numerically are shown in Figure [4.5] and [4.6]. During computation procedures, 100 grids and 30 grids each are used to represent the total length of 100 meters for the system.

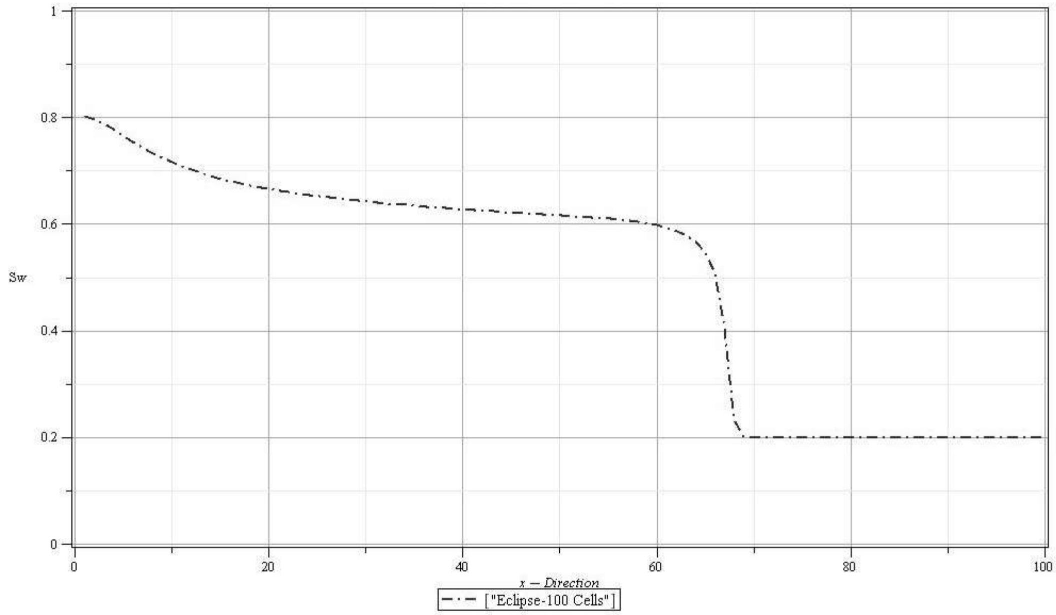


Figure (15): Saturation Profiles Calculated from Numerical Computations by using Eclipse for 100 Cells after 30 days of Water Injection.

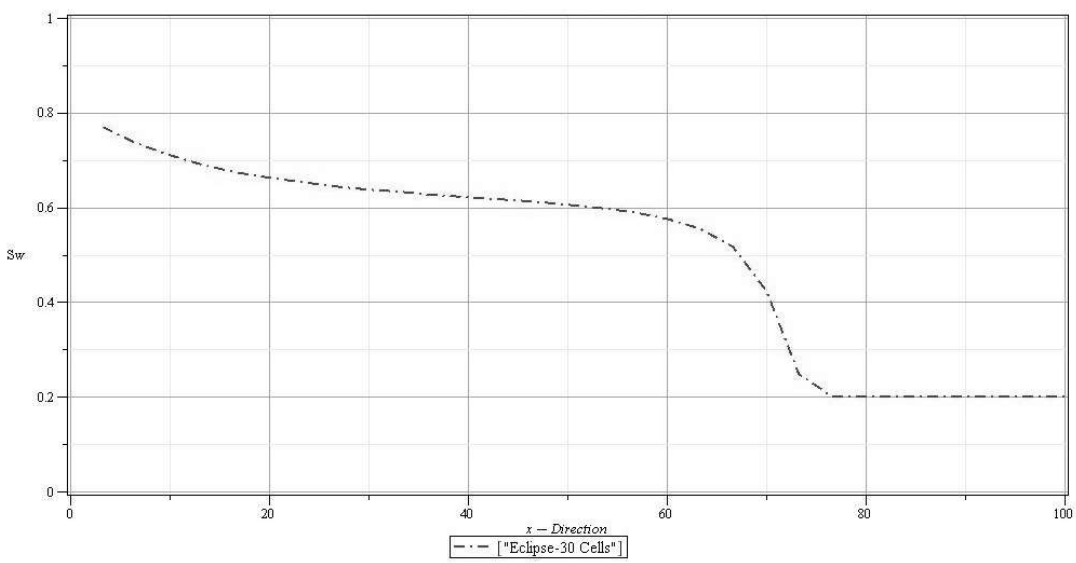


Figure (16): Saturation Profiles Calculated from Numerical Computations by using Eclipse for 30 Cells after 30 days of Water Injection.

The water is injected into the reservoir with a linear flow rate of 1 m^3/day. The oil and water viscosities are 0.001 (Pa s) and 0.004 (Pa s), respectively. The flow of the displaced phase (oil) ceases at S_{or} of 0.15. The porosity of the medium is 20% with an absolute permeability of k = 1 Darcy.

7.3. Grid Sensitivity

For the sake of brevity, we will show two runs that we have performed: fine – and coarse grid runs. Fine grid runs are performed by using 100 grids while the coarse grid runs are performed by using 30 grids, as shown in Figure [17].

For the sake of comparison, the figure below presents the Analytical and the Numerical Solution of Buckley Leverett.

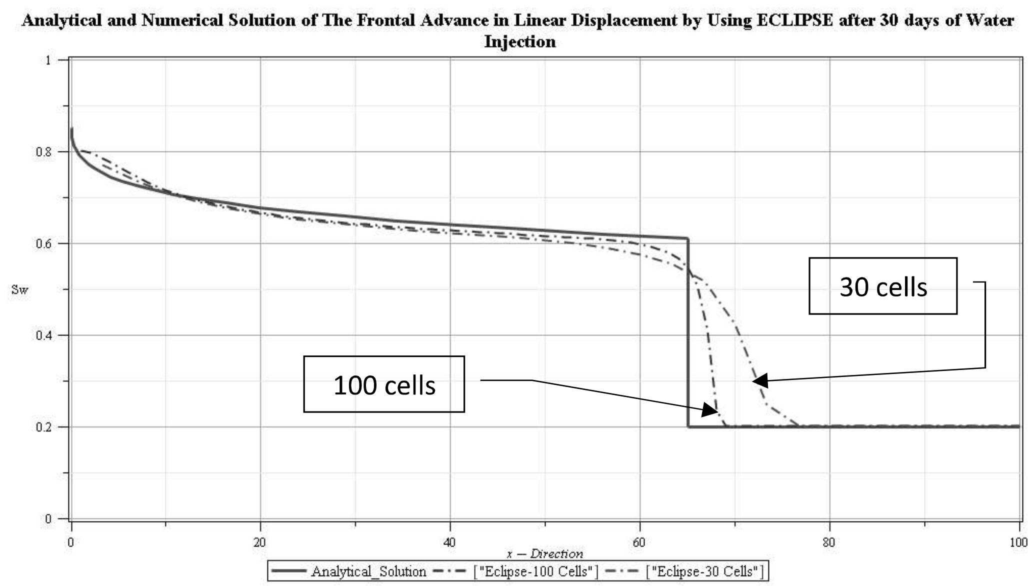


Figure (17): Analytical and numerical solution of the frontal advance in linear displacement.

Comparison of the Numerical Solutions to Buckley-Leverett Results For the purpose of convenience and easy comparison, the numerical solutions and Buckley-Leverett results, which represent the saturation profiles along the distance X, are presented on the same plot for various times.

Figures [18] through [20] contain the data curves separately for various times of water injection (1 day, 5 days, 15 days, 30 days). With the exception of locations near the water injection point and at the displacement front.

These comparisons show that the numerical solutions in each case are in good agreement with solutions obtained from the Buckley-Leverett method. They are of satisfactory accuracy for most engineering calculations. The only discrepancy in the numerical solutions is a very slight smearing of the displacement front.

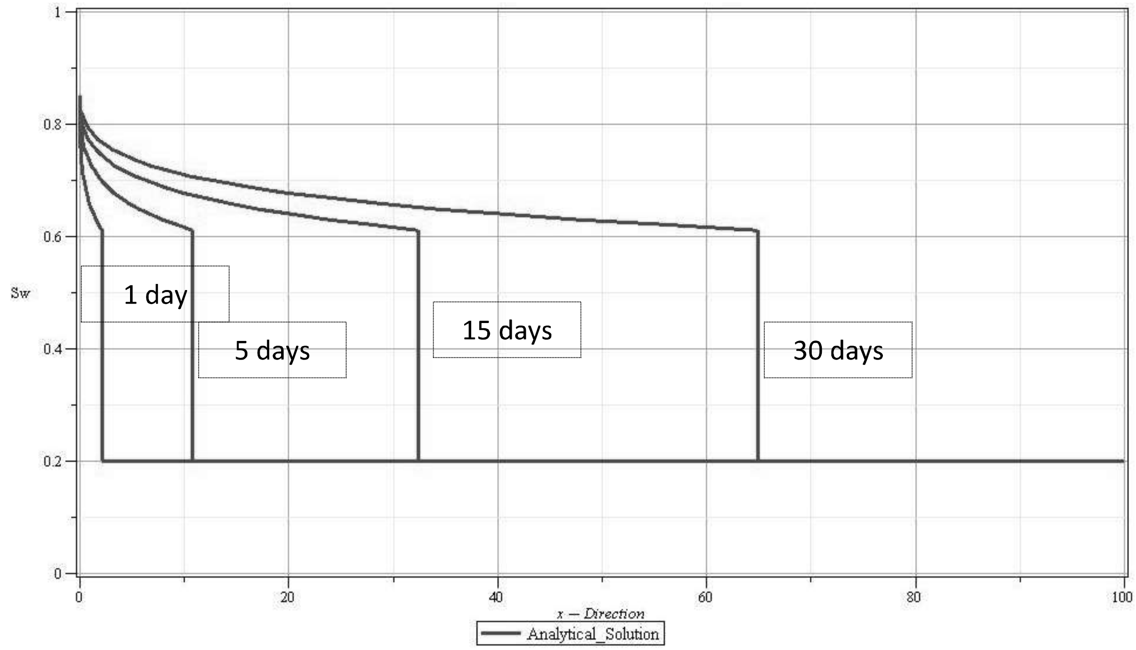


Figure (18): Distribution of water saturation profile in the analytical solution at different times.

The numerical solutions were obtained with the single-point upstream method, where the average forward fluid mobility is calculated from the relative permeability and viscosity data computed using fluid saturation and pressure in block i (the upstream block).

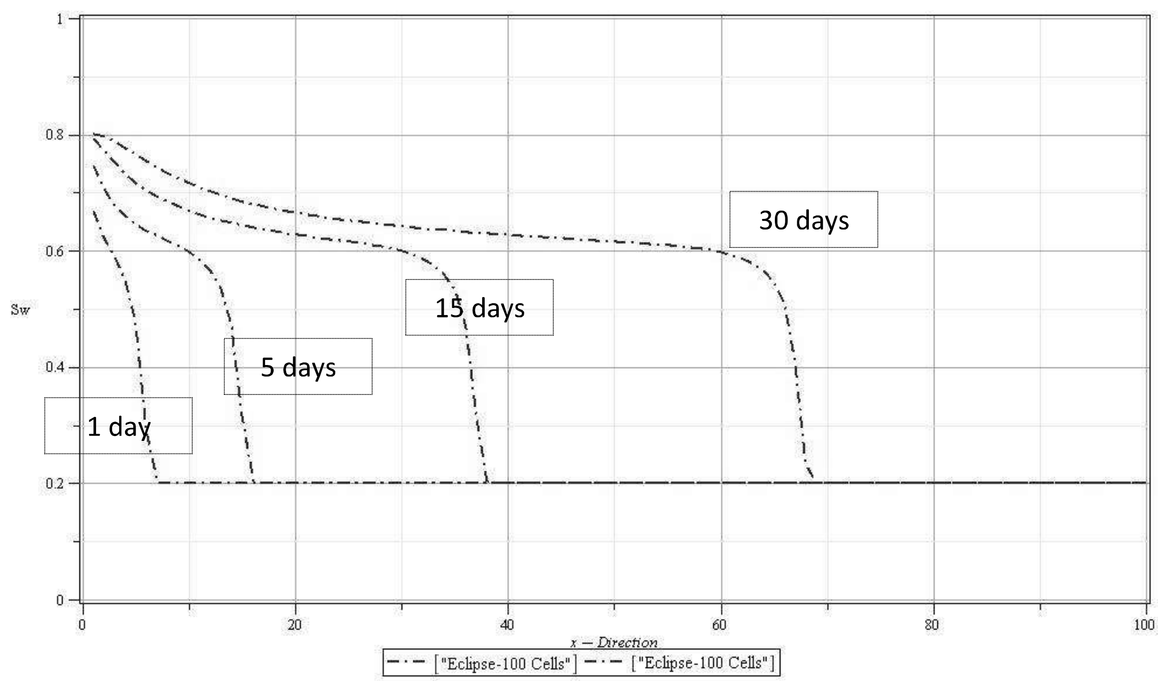


Figure (19): Distribution of water saturation profile in the numerical solution at different times.

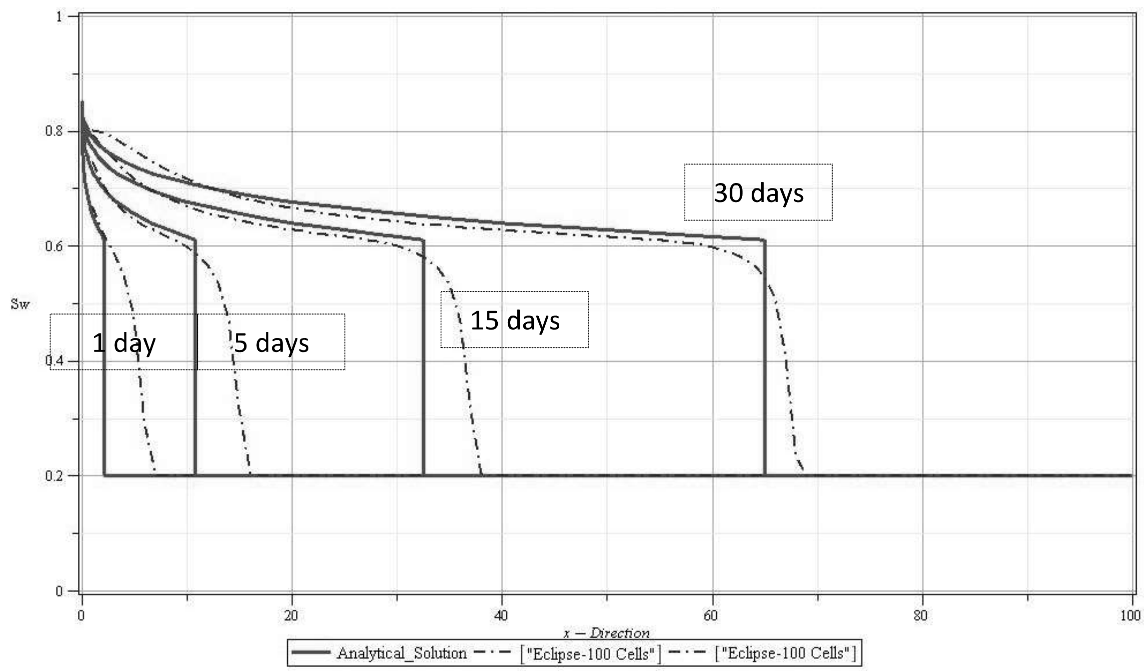


Figure (20): Analytical and Saturation Profiles Calculated from Numerical Computations by using Eclipse for 100 Cells at different times.

8. Conclusion:

1. In most cases, the fluid flow inside the porous rock is too complicated to solve analytically. These methods can apply to some simplified models. However, this solution can be applied as the benchmark solution to validate the numerical approaches.
2. The method represented here is limited to a one-dimensional, two-phase liquid reservoir system. However, this is adequate for modeling many important mechanisms of reservoir drive, or secondary recovery methods, such as waterflooding.
3. The numerical computation procedures for the analysis have been developed and programmed by Eclipse 100 Model for 100 and 30 cells.
4. The treatment of immiscible displacement is by no means exhaustive. The main objective was to introduce the engineer to basic concepts and some of the historical theoretical developments in immiscible displacements. In the process, several important terms were introduced in discussing the importance of rock wettability, capillary pressure, relative permeability, mobility ratio, and displacement efficiency in immiscible displacements. The fractional flow equation was developed to convey the impact of fluid and rock properties and reservoir geometry on immiscible displacement. In fact, this relatively simple equation can be used to discuss the fractional flow of water or gas in many waterflood or gasflood projects.

5. The Welge method was introduced to illustrate determination of several terms, such as saturations at the flood front, average saturations behind the flood front, saturations at breakthrough, and saturations after breakthrough.
6. The analytical model is developed for applying the Buckley-Leverett frontal advance theory to immiscible displacement in 1-Dimension for a length of 100 meter, neglecting the capillary forces. The analytical model gives more accurate results as compared to conventional models.
7. The saturation profiles, which were calculated, are shown to be in satisfactory agreement with the Buckley-Leverett results. The fine model of 100 cells shows better results than the 30 cells in comparison with the analytical one, where the forward fluid mobility is based on the relative permeability and viscosity data.

9. Recommendation:

Relative permeability curves are very significant in any study of fluid flow. Accordingly, experimental measurement of relative permeability curves needs to be carried out.

10. References

1. Buckley, S.E. and Leverett, M.C. "Mechanism of Fluid Displacement in Sands," Transactions ATME. (1942) Vol. 146. 115.
2. Douglas, J.Jr., Blair, P.M. and Wagner, R.J. "Calculation of Linear Waterflood Behavior Including the Effects of Capillary Pressure," Transaction AIME. (1958) Vol. 213.
3. Holmgren, C.A. (1951). Effect of free gas saturation on oil recovery by waterflooding. Trans. AI ME, vol. 192, pp. 135-140.
4. West, W.J., Garvin. W.W. and Sheldon, J.W. "Solution of the Equations of Unsteady State, Two-Phase Flow in Oil Reservoirs," Transactions AIME. (1954). Vol. 201.
5. Douglas, J.Jr., Blair, P.M. and Wagner, R.J. "Calculation of Linear Waterflood Behavior Including the Effects of Capillary Pressure," Transaction AIME. (1958) Vol. 213.
6. Fayers, F.J. and Sheldon, J.W. "The Effect of Capillary Pressure and Gravity on Two-Phase Fluid Flow in a Porous Medium," Transactions ATMF. (1959). Vol. 216. 147-155.
7. Eclipse Technical Description, version 2009.1, Schlumberger.
8. Zhangxin Chen, 2007. Reservoir Simulation Mathematical Techniques in Oil Recovery. SIAM, Philadelphia.
9. Craig, F.F. Jr., "Reservoir Engineering Aspects of Waterflooding," Monograph Series, SPE, 1971.
10. Essley, P.L., Jr., Hancock, G.L., Jr., and Jones, K.E.: "Gravity Drainage Concepts in a Steeply Dipping Reservoir," paper SPE 1029-G, SPE of AIME, Petroleum Conference on Production and Reservoir Engineering, Tulsa, Oklahoma, March 20–21, 1958
11. Nnaemeka Ezekwe, 2010. Petroleum Reservoir Engineering Practice. Prentice Hall, Boston.
12. Welge, H.J.: "A Simplified Method for Computing Oil Recovery by Gas or Water Drive, "Trans. AIME (1952) 195, 91–98.

13. Liu, J.S., Wilkins, J.R., Al-Qahtani, M.Y., and Al-Awami, A.A.: "Modeling a Rich Gas Condensate Reservoir with Composition Grading and Faults," paper SPE 68178, presented at the SPE Middle East Oil Show, Bahrain, Mar. 17-20, 2001.
14. Willhite, G.P.: Waterflooding, SPE Textbook Series, Vol. 3, Richardson, Texas (1986).
15. Treiber, L.E., Archer, D.L., and Owens, W.W.: "A Laboratory Evaluation of the Wettability of Fifty Oil-Producing Reservoirs," SPEJ (Dec. 1972) 531–540.
16. www.ipims.com